CHAPTER 9

Analyses for Rapid Drawdown

Rapid drawdown takes place when the water level outside a slope drops so quickly that impermeable soils within the slope do not have sufficient time to drain. As the water level drops, the stabilizing effect of the water outside the slope is removed, and the shear stresses for equilibrium increase. The shear stresses within the slope are resisted by undrained strength in zones of low permeability and by drained strength within zones of higher permeability. This is a severe loading condition that can cause failure of slopes that are stable before drawdown.

Whether a soil zone drains or not can be estimated by calculating the value for the dimensionless time factor, T, given by

$$T = \frac{c_v\, t}{D^2} \tag{9.1}$$

where c_v is the coefficient of consolidation, t the time for drawdown, and D the drainage distance. Typical values of c_v for various soils are shown in Table 9.1. If the calculated value of T is equal to 3 or more, the dissipation of pore water pressures induced by the drawdown exceeds 98%, and it is reasonable to treat the material as drained. Most soils with coefficients of permeability of 10^{-4} cm/s or more can be assumed to drain under normal rates of drawdown, and drained shear strengths can be used for these zones.

Rapid drawdown may occur at any time during the life of a slope, including during construction if the slope is built in water or water rises next to the slope during construction. Therefore, it may be necessary to analyze rapid drawdown for during and end-of-construction stability as well as for long-term conditions. The approaches and shear strengths used for the two cases (during and end of construction; long term) are different and are described in the following sections.

DRAWDOWN DURING AND AT THE END OF CONSTRUCTION

If drawdown occurs during or immediately after construction, the appropriate shear strengths to be used in stability computations are the same as those used when no drawdown occurs: For soils that drain freely, the shear strengths are expressed in terms of effective stresses and appropriate pore water pressures are used. For soils that do not drain freely, undrained shear strengths, determined from the results of unconsolidated–undrained shear tests, are used. Stability computations are performed using effective stresses for soils that drain and total stresses for soils that do not drain.

DRAWDOWN FOR LONG-TERM CONDITIONS

If drawdown occurs long after construction, the soils within the slope will have had time to come to equilibrium under a new effective stress regime. The undrained shear strengths of low-permeability soils during drawdown are governed by the consolidation stresses in the equilibrium state before drawdown. The drained shear strengths of high-permeability soils are governed by the water pressures after drawdown.

Stability at the end of rapid drawdown has been analyzed in two basically different ways: (1) using effective stress methods, and (2) using total stress methods, in which the undrained shear strengths of low-permeability soils are related to effective consolidation pressures in the slope prior to drawdown. Both methods treat free-draining materials in the same way. The

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Table 9.1 Typical Values of the Coefficient of Consolidation, c_v

Type of soil	Values of c_v (ft^2/day)
Coarse sand	>10,000
Fine sand	100–10,000
Silty sand	10–1000
Silt	0.5–100
Compacted clay	0.05–5
Soft clay	<0.2

strengths of free-draining materials are expressed in terms of effective stresses, and the pore water pressures are estimated assuming either steady seepage or hydrostatic conditions depending on the particular slope.

Effective Stress Methods

The advantage of effective stress analyses is that it is relatively easy to evaluate the required shear strength parameters. The effective stress shear strength parameters for soils are readily determined by means of isotropically consolidated undrained (IC-U) triaxial compression tests with pore pressure measurements. This type of test is well within the capability of most soil mechanics laboratories.

The disadvantage of effective stress analyses is that it is difficult to estimate the pore water pressures that will exist within low-permeability soils during drawdown. The pore pressure changes during drawdown depend on the changes in stress that result from the changing water loads on the slope and the undrained response of the soils within the slope to these changes in load. While the changes in stress can be estimated with reasonable accuracy, particularly at shallow depths beneath the surface of the slope, the undrained response of the soil is much harder to estimate. The changes in pore pressure are considerably different for materials that tend to dilate during shear and those that do not. Although in principle it is possible to estimate these pore water pressures, for example, by using Skempton's pore water pressure coefficients (Skempton, 1954), in practice this is difficult and the results are uncertain.

Most effective stress analyses of stability during rapid drawdown have used the assumptions regarding pore water pressures that were suggested by Bishop (1954) and later used by Morgenstern (1963). These assumptions have been justified on the basis of the fact that they are conservative in most cases. They have been found to result in reasonable values of factor of safety for dams that suffered rapid drawdown failures: Wong et al. (1983) found that the values of safety factor calculated using Morgenstern's assumption were $F = 1.2$ for Pilarcitos Dam and $F = 1.0$ for Walter Bouldin Dam, both of which failed.

It seems likely that Bishop and Morgenstern's assumptions for pore water pressures during drawdown may be more accurate for soils that do not tend to dilate during shear than for those that do tend to dilate. Thus, although these assumptions may show reasonable correspondence with failures of slopes in materials that are not densely compacted and do not tend to dilate during shear, they are likely to be unduly conservative for better-compacted materials that do tend to dilate during shear. Use of effective stress analyses based on the Bishop and Morgenstern assumptions would treat all fill materials alike with respect to pore water pressures during drawdown, regardless of how well they are compacted or how strongly they might tend to dilate during shear.

Terzaghi and Peck (1967) suggested that pore water pressures during drawdown in well-compacted silty sands could be estimated using flow nets. Several investigators (Browzin, 1961; Brahma and Harr, 1963; Newlin and Rossier, 1967; Desai and Sherman, 1971; Desai, 1972, 1977) used theoretical methods to analyze the nonsteady flow conditions following drawdown. Like the Bishop and Morgenstern pore pressure assumptions, these methods do not consider the behavior of the soil with regard to dilatancy and are thus not capable of representing all of the important factors that control the pore water pressures during drawdown.

Svano and Nordal (1987) and Wright and Duncan (1987) used procedures for estimating pore water pressures during drawdown that reflect the influence of dilatancy on the pore water pressure changes. Svano and Nordal used two-stage stability analyses and iterated to achieve consistency between the calculated factor of safety and the values of the pore water pressures. Wright and Duncan used finite element analyses to estimate the stress changes during drawdown and Skempton's pore pressure parameters to calculate the pore water pressures. These studies indicate that it is possible to estimate realistic pore water pressures for effective stress analyses, but it is more cumbersome and difficult than using total stress analyses, as described in the following sections.

Terzaghi and Peck (1967) summarized the problems in estimating pore water pressures during drawdown in these words:

> [I]n order to determine the pore pressure conditions for the drawdown state, all the following factors need to be known: The location of the boundaries between materials

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> with significantly different properties; the permeability and consolidation characteristics of each of these materials; and the anticipated maximum rate of drawdown. In addition, the pore pressures induced by the changes in the shearing stresses themselves . . . need to be taken into consideration. In engineering practice, few of these factors can be reliably determined. The gaps in the available information must be filled by the most unfavorable assumptions compatible with the known facts.

By using undrained strengths in low-permeability zones, as described in the following sections, many of the problems associated with estimating pore water pressures for effective stress analyses can be avoided, and the accuracy of rapid drawdown stability analyses can be improved very considerably.

Total Stress Methods

Total stress methods are based on undrained shear strengths in low-permeability zones. The undrained shear strengths are estimated based on the effective stresses that exist in the slope prior to drawdown. Some zones within the slope may consolidate with time following construction, and their undrained strengths will increase. Portions of the same soils at lower stresses (near the surface of the slope) may expand following construction, and their undrained strengths will decrease with time.

Several total stress analysis methods have been suggested and used for sudden-drawdown analyses. These include the U.S. Army Corps of Engineer's (1970) method, and Lowe and Karafiath's (1959) method. Duncan et al. (1990) reviewed both of these methods and suggested an alternative *three-stage analysis* procedure that is described in the following paragraphs. The three-stage procedure combines the best features of both the Corps of Engineers' and Lowe and Karafiath's methods. Following along the lines of the Corps of Engineers' method, the three-stage procedure accounts for the effect of drainage and the fact that the drained strength may be less than the undrained strength. It differs from the Corps of Engineers' procedure in the way that the undrained strength is evaluated and the way that the drained strength is taken into account. Following Lowe and Karafiath's suggestion, the three-stage procedure accounts for the effects of anisotropic consolidation, which can result in significantly higher undrained shear strength.

Each stage in the three-stage stability analysis procedure involves a separate set of slope stability computations for each trial slip surface. For free-draining materials, effective stresses are used for all three stages, with different pore water pressures based on water levels and seepage conditions. The effective stress shear strength parameters are the same for all three stages. For low-permeability zones effective stresses are used for the first stage, before drawdown, and total stresses and undrained strengths are used for the second stage, after drawdown. For the third stage the lower of the drained and undrained strengths is used, whichever is lower, to be conservative.

First-stage computations. The first-stage stability computations are performed for conditions prior to drawdown. The purpose of the computations is to estimate effective stresses along the slip surface prior to drawdown. Effective stress shear strength parameters are used with pore water pressures based on estimated groundwater and seepage conditions. Steady-state seepage is assumed. Although the first-stage stability computations are performed in the same way that long-term stability computations are performed, the purpose is not to calculate the factor of safety; only the effective stresses on the slip surface are of interest.

The first-stage stability computations are used to calculate the shear stress and effective normal stress on the slip surface. The effective normal stress (σ') is computed for the base of each slice from the total normal force (N) on the bottom of each slice and the corresponding pore water pressure:

$$\sigma'_{fc} = \frac{N}{\Delta l} - u \tag{9.2}$$

This normal stress represents the effective stress on a potential *failure* plane that exists at the time of *consolidation.* Accordingly, this normal stress and the corresponding shear stress are designated with the subscript fc. The corresponding shear stress is computed from the Mohr–Coulomb equation and the factor of safety using

$$\tau_{fc} = \frac{1}{F}(c' + \sigma' \tan \phi') \tag{9.3}$$

or, alternatively, the shear stress can be computed from the shear force on the base of each slice from the relationship

$$\tau_{fc} = \frac{S}{\Delta l} \tag{9.4}$$

where S is the shear force on the base of the slice. These consolidation stresses (σ'_{fc} and τ_{fc}) are used to estimate undrained shear strengths for the second-stage computations.

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Second-stage computations. In the second-stage computations, undrained shear strengths and total stress analysis procedures are used for low-permeability zones. Undrained shear strengths are estimated for the second stage using the stresses calculated from the first stage and shear strength envelopes that relate the undrained shear strength to the effective consolidation pressure. The undrained shear strength is expressed as the value of the shear stress on the failure plane at failure, τ_{ff} (Fig. 9.1). The subscript ff distinguishes this value of shear stress from the value of shear stress at consolidation (τ_{fc}). The shear stress on the failure plane at failure is calculated from the principal stress difference, $\sigma_1 - \sigma_3$, at failure and the friction angle, ϕ', using the relationship

$$\tau_{ff} = \frac{\sigma_{1f} - \sigma_{3f}}{2} \cos \phi' \qquad (9.5)$$

The effective stress friction angle is the same one used for the first-stage analysis where effective stress envelopes are used for all soils. If the friction angle varies with effective stress, the value of the friction angle for the applicable range of effective stresses is used.

Undrained shear strength for second stage. The undrained shear strength, τ_{ff}, is plotted versus the effective stress on the failure plane at consolidation, σ'_{fc}. Two shear strength envelopes are plotted (Fig. 9.2): One corresponds to isotropic consolidation ($K_c = \sigma'_{1c}/\sigma'_{3c} = 1$), the other corresponds to anisotropic consolidation with the maximum effective principal stress ratio possible (i.e., $K_c = K_{\text{failure}}$). The first envelope ($K_c = 1$) is obtained from consolidated–undrained triaxial shear tests with isotropic consolidation. The effective stress on the failure plane (σ'_{fc}) for this envelope is the isotropic consolidation pressure, σ'_{3c}. The envelope for $K_c = 1$ is obtained by plotting τ_{ff} calculated from Eq. (9.5) versus the effective consolidation pressure, σ'_{3c}. The second shear strength envelope ($K_c = K_f$) is the same as the effective stress envelope used in the first-stage stability computations. The intercept and slope of the strength envelope for $K_c = K_f$ are the same as the effective cohesion and friction angle, c' and ϕ', respectively. This envelope corresponds to the maximum effective principal stress ratio possible, with the soil at failure during consolidation.

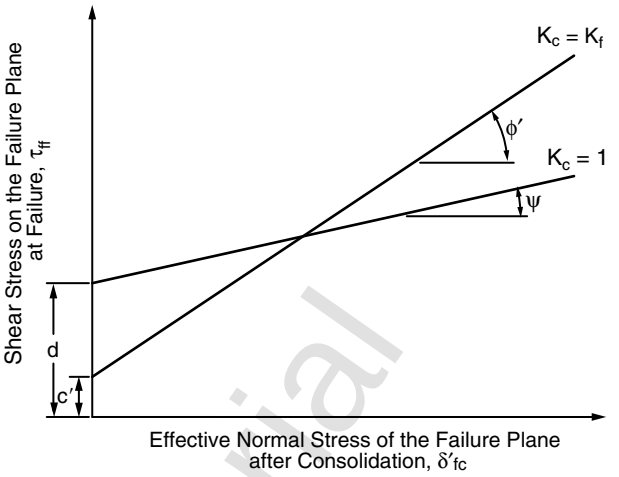


Figure 9.2 Shear strength envelopes used to define the undrained shear strengths for the second stage of a three-stage analysis.

The slope and intercept of the strength envelope for $K_c = 1$ are related to the intercept and slope of the failure envelope that is often referred to as the R or *total stress envelope.* The R envelope is drawn on a Mohr diagram by plotting circles where σ_3 is the minor principal stress at consolidation (σ'_{3c}) and the principal stress difference (diameter of circle) is equal to the principal stress difference at failure, $\sigma_{1f} - \sigma_{3f}$, as illustrated in Fig. 9.3.[1] The intercept and slope of the failure envelopes drawn on such a diagram are designated c_R and ϕ_R. The intercepts and slopes for the τ_{ff} vs. σ'_{fc} envelope and the R envelope are similar but not equal. However, the intercepts and slopes of the two envelopes can be related to one another. If c_R and ϕ_R are the respective intercept and slope angle for the R envelope and the envelope is drawn tangent to the circles on a Mohr diagram (Figure 9.3*a*), the intercept, d,

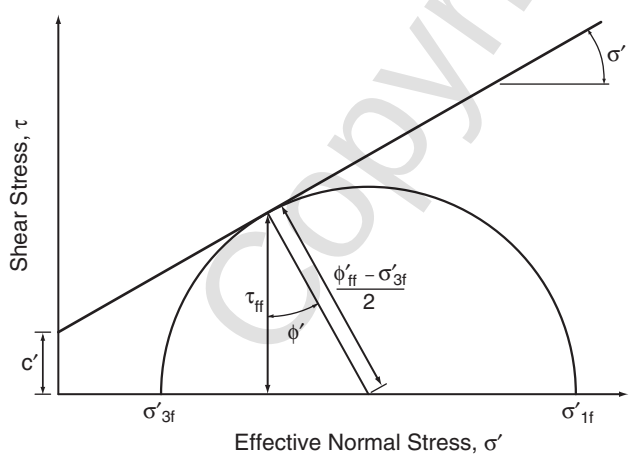


Figure 9.1 Mohr's circle for effective stresses at failure showing the shear stress on the failure plane.

[1] *Note:* These are not actually Mohr's circles because one stress (σ'_{3c}) is at consolidation while the other stress ($\sigma_{1f} - \sigma_{3f}$) is at failure.

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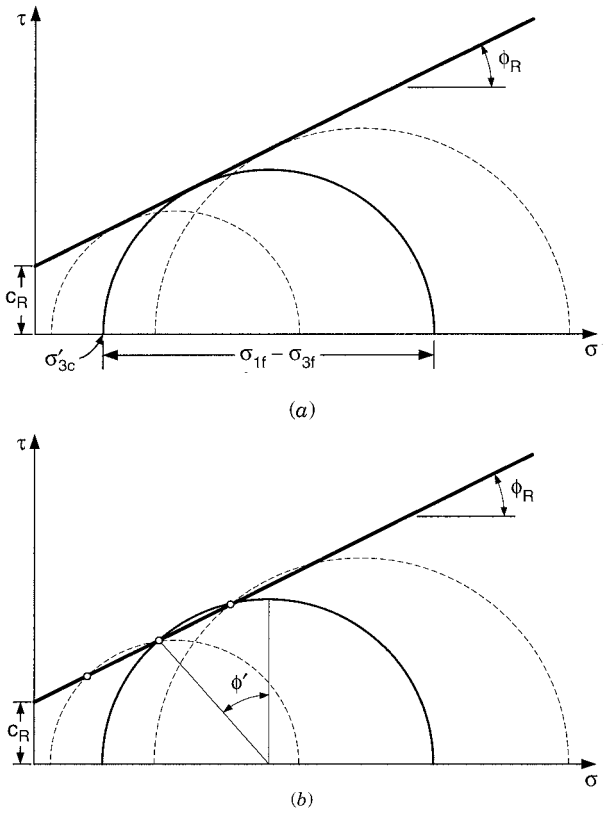


Figure 9.3 Shear strength envelopes used to define the undrained shear strengths for the second stage of three-stage analyses: (*a*) failure envelope tangent to circles; (*b*) failure envelope through points representing stresses on the failure plane.

and slope, ψ, for the corresponding $K_c = 1$ envelope are related as follows:

$$d_{K_c=1} = c_R \frac{\cos \phi_R \cos \phi'}{1 - \sin \phi_R} \tag{9.6}$$

$$\psi_{K_c=1} = \arctan \frac{\sin \phi_R \cos \phi'}{1 - \sin \phi_R} \tag{9.7}$$

The *R* envelope is sometimes drawn such that it passes through points corresponding to stresses on the failure plane. If the *R* envelope is drawn in this manner (Figure 9.3*b*), the slope and intercept of the $K_c = 1$ envelope are computed from

$$d_{K_c=1} = \frac{c_R}{1 + (\sin \phi' - 1) \tan \phi_R / \cos \phi'} \tag{9.8}$$

$$\psi_{K_c=1} = \arctan \frac{\tan \phi_R}{1 + (\sin \phi' - 1) \tan \phi_R / \cos \phi'} \tag{9.9}$$

Variation of τ_{ff} with σ'_{fc} and K_c. The undrained shear strength envelopes shown in Figure 9.2, for $K_c = 1$ and $K_c = K_f$, represent the extremes possible for the lowest and highest possible values of K_c. Lowe and Karafiath (1959) showed that for the same value of σ'_{fc}, the undrained strength (τ_{ff}) varies with the value of K_c. They recommended that anisotropically consolidated undrained (ACU) triaxial tests be performed to measure undrained shear strengths using a range of K_c values, to develop data that could be used to evaluate undrained strengths for second-stage analyses. This procedure results in accurate evaluation of undrained strengths for the second-stage analyses but requires extensive and difficult testing. The ACU test specimens must be consolidated slowly to avoid failure during consolidation.

Wong et al. (1983) found that the difficult ACU tests could be avoided by interpolating undrained strengths for values of K_c between $K_c = 1$ and $K_c = K_f$ instead of determining the strengths experimentally. They found that values of τ_{ff} calculated assuming that τ_{ff} varies linearly with K_c were the same as measured values. Using this method of interpolation, only isotropically consolidated undrained (ICU) tests with pore water pressure measurements need be performed. Both of the envelopes shown in Figure 9.2 can be determined from these tests, which are much easier to perform than ACU tests.

Once the $K_c = K_f$ and $K_c = 1$ envelopes shown in Figure 9.2 have been determined, the undrained shear strength for the second-stage computations is obtained based on the effective normal stress on the slip surface, σ' [Eq. (9.2)] and the estimated effective principal stress ratio for consolidation, K_c. The effective principal stress ratio for consolidation is estimated following the recommendations of Lowe and Karafiath (1959). They suggested the assumption that the orientation of the principal stresses at consolidation is the same as the orientation of the principal stresses at failure. This leads to the following equation for the effective principal stress ratio:

$$K_1 = \frac{\sigma' + \tau[(\sin \phi' + 1)/\cos \phi']}{\sigma' + \tau[(\sin \phi' - 1)/\cos \phi']} \tag{9.10}$$

where K_1 is the effective principal stress ratio for consolidation corresponding to the stage 1 analyses and σ' and τ are the effective normal stress and shear stress on the shear plane at consolidation.

Values of the effective principal stress ratio can be calculated from Eq. (9.10) using the shear stress and effective normal stress calculated from Eqs. (9.2) and (9.3); the friction angle is the effective stress friction angle used in the first-stage computations and in Eq.

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(9.5) to calculate τ_{ff}. Values of the undrained shear strength for the effective consolidation pressure, σ', and a consolidation stress ratio $K_c = K_1$ are obtained from the two shear strength envelopes shown in Figure 9.2. Linear interpolation between the values for $K_c = 1$ and $K_c = K_f$ to obtain the value corresponding to K_1 can be expressed by

$$\tau_{ff} = \frac{(K_f - K_1)\tau_{ff(K_c=1)} + (K_1 - 1)\tau_{ff(K_c=K_f)}}{K_f - 1} \quad (9.11)$$

where $\tau_{ff(K_c=1)}$ and $\tau_{ff(K_c=K_f)}$ are the undrained shear strengths from the two shear strength envelopes shown in Figure 9.2. The undrained shear strengths are determined from these two envelopes using the value of the effective stress, σ'_{fc}, which was calculated from Eq. (9.2).

The effective principal stress ratio at failure shown in Eq. (9.11) can be calculated from the effective stress shear strength parameters. If there is no cohesion, the value for K_f does not depend on the magnitude of the stress and is given by

$$K_f = \tan^2\left(45 + \frac{\phi'}{2}\right) \quad (9.12)$$

If the value of c' is not zero, the value of K_f depends on the effective stress on the slip surface and is given by

$$K_f = \frac{(\sigma' + c' \cos \phi')(1 + \sin \phi')}{(\sigma' - c' \cos \phi')(1 - \sin \phi')} \quad (9.13)$$

where σ' is the effective stress on the slip surface after consolidation (and also at failure because $K_c = K_f$).

If a significant effective cohesion (c') exists, the effective minor principal stress, σ'_3 implicit in Eqs. (9.10) and (9.13) may become negative (i.e., the terms in the denominator of these equations becomes negative). When this occurs, the effective consolidation stress ratios become negative and are nonsensical. The negative stress results from the fact that the soil is assumed to have *cohesion,* and this implies a tensile strength. The corresponding negative values for σ'_3 are, however, not realistic. In cases where negative effective stresses are calculated, the values are rejected and instead of interpolating shear strengths using values of K_c, the shear strength is taken to be the lower of the shear strengths from the $K_c = 1$ and $K_c = K_f$ envelopes. Negative effective stresses can be detected by calculating the effective minor principal stresses after consolidation using Eqs. (9.14) and (9.15):

$$\sigma'_{3c} = \sigma'_{fc} + \tau_{fc} \frac{\sin \phi' - 1}{\cos \phi'} \quad (9.14)$$

$$\sigma'_{3f} = (\sigma'_{fc} - c' \cos \phi') \frac{1 - \sin \phi'}{\cos^2 \phi'} \quad (9.15)$$

Equation (9.14) is the effective minor principal stress corresponding to the $K_c = 1$ envelope; Eq. (9.15) corresponds to the $K_c = K_f$ envelope. The values of σ'_{3c} and σ'_{3f} correspond to the stresses in Eqs. (9.10) and (9.13), respectively. If either value is negative (or zero), no interpolation is performed and the lower of the $K_c = 1$ and $K_c = K_f$ strengths is used for the undrained shear strength in the second-stage stability computations. After the undrained shear strength is determined for each slice, a total stress analysis is performed using the undrained shear strengths and the external water loads after drawdown.

The factor of safety calculated in the second-stage analysis assumes that all of the low-permeability materials are undrained during rapid drawdown. Additional, third-stage computations are performed to check if the drained shear strength might be lower than the undrained shear strength, and thus the factor of safety would be lower if these low-permeability materials were drained rather than undrained.

Third-stage computations. For the third stage the undrained shear strengths used for the second-stage computations are compared with the drained strengths for each point along the slip surface. The drained shear strengths are estimated using the total normal stresses from the second-stage analysis and pore water pressures corresponding to the water level after drawdown. The total normal stresses are calculated for each slice using

$$\sigma = \frac{N}{\Delta l} \quad (9.16)$$

where N is the total normal force on the base of the slice calculated for the second stage and Δl is the length of the base of the slice. The effective stress is the total normal stress minus the applicable pore water pressure, u. The drained strength that would exist is then calculated from the Mohr–Coulomb equation,

$$s = c' + (\sigma - u) \tan \phi' \quad (9.17)$$

If at any point on the slip surface the drained shear strength calculated using Eq. (9.17) is lower than the undrained shear strength, an additional slope stability calculation is performed. If, however, the estimated drained strengths are all higher than the undrained

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shear strengths used for the second-stage computations, no additional computations are performed, and the factor of safety calculated from the second-stage computations is the factor of safety for rapid drawdown.

When a third set of stability computations is performed, new shear strengths are assigned to the slices where the drained shear strengths were estimated[2] to be lower than the undrained shear strengths. For these slices effective stress shear strength parameters are assigned and appropriate pore water pressures are stipulated. If the estimated drained strength for any slice is greater than the undrained shear strength, the strength is not changed, and undrained shear strength is used for the third stage. Thus, some portions of the slip surface will have effective stress shear strength parameters assigned and others will still have undrained shear strengths. Once the appropriate drained or undrained shear strength has been assigned for each slice, the third-stage stability calculations are performed. The factor of safety from the third-stage calculations represents the factor of safety for rapid drawdown. If the third-stage calculations are not required (i.e., undrained shear strengths are less than drained shear strengths for low-permeability materials everywhere along the slip surface), the factor of safety from the second stage is the factor of safety for rapid drawdown, as mentioned previously.

Example. A simple example problem is described to illustrate the three-stage analysis procedure described above. To simplify the calculations and allow them to be performed easily by hand, the example considers the rapid drawdown stability of an infinite slope. The slope is 3 (horizontal):1 (vertical), as shown in Figure 9.4. At the point in the slope where the stability calculations are performed, the slope is assumed to be submerged beneath 100 ft of water before drawdown. Although a depth of water is assumed, the water depth has no effect on the final results as long as the slope is fully submerged before drawdown. The total (saturated) unit weight of the soil is 125 pcf. The effective stress shear strength parameters are $c' = 0$, $\phi' = 40°$. The intercept and slope of the $K_c = 1$ undrained shear strength envelope (d and ψ) are 2000 psf and 20°. The shear strength envelopes are shown in Figure 9.5.

[2] At the end of the second stage the drained shear strengths can only be estimated because the normal stress that corresponds to drained shear strengths depends to some extent on the drained strengths themselves. The drained strengths calculated at the end of the second stage are based on total normal stresses that were calculated using undrained rather than drained shear strengths. Thus, the drained strengths are considered to be "estimated" on the basis of the second stage. However, the degree of approximation involved in this estimate is very small.

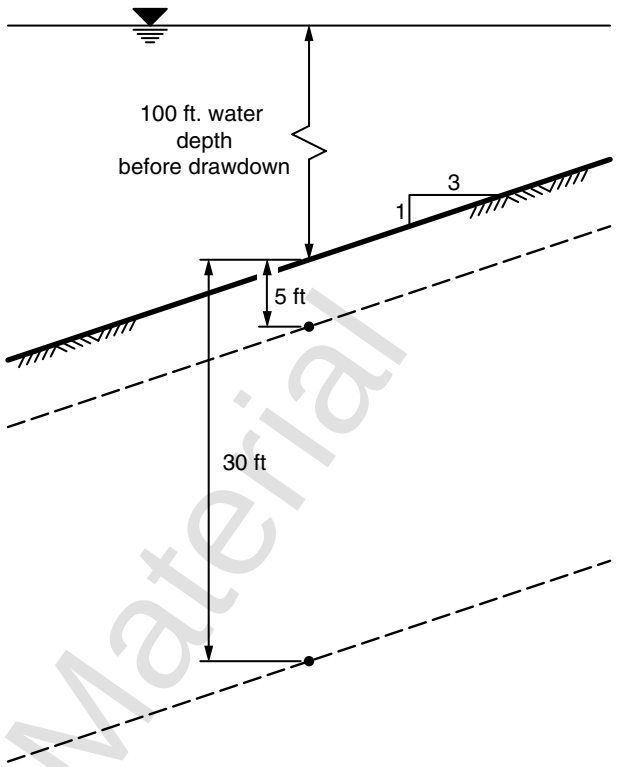


Figure 9.4 Slope used for example calculations of stability due to rapid drawdown using three-stage analysis procedure.

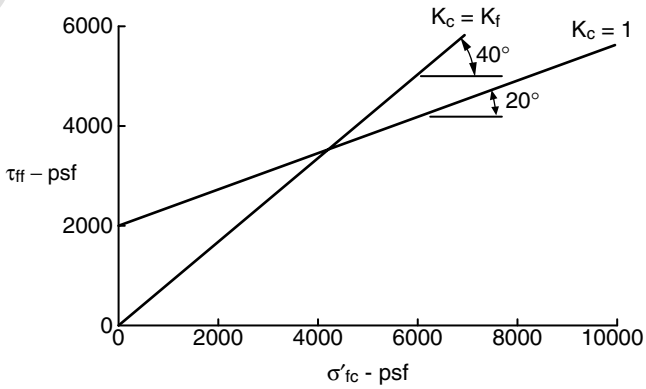


Figure 9.5 Shear strength envelopes for example problem.

Stability calculations are performed for two slip surfaces. Both of the slip surfaces are planes parallel to the surface of the slope; one is 5 ft deep, the other is 30 ft deep. Total drawdown is assumed to occur (i.e., the water is assumed to be fully removed from the face of the slope). It is further assumed that after drainage has occurred, the pore water pressures are zero. The

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calculations for each of the three stages are described below, and key quantities are summarized in Table 9.2.

First-stage analysis. The total normal stress on the slip surface is computed for a submerged infinite slope from the equation

$$\sigma = \gamma z \cos^2\beta + \gamma_w(h_w + z \sin^2\beta) \qquad (9.18)$$

Thus, for the slip surface at a depth of 5 ft,

$$\sigma = (125)(5)\cos^2(18.4°) + (62.4)[100 + (5)\sin^2(18.4°)] = 6834 \text{ psf} \qquad (9.19)$$

Similarly, for the slip surface at a depth of 30 ft, the total normal stress is 9803 psf. The pore water pressure on the slip plane is computed from

$$u = \gamma_w(z + h_w) \qquad (9.20)$$

For the slip surface at a depth of 5 ft, the pore water pressure is

$$u = (62.4)(5 + 100) = 6552 \text{ psf} \qquad (9.21)$$

and similarly, the pore water pressure at a depth of 30 ft is $u = 8112$ psf. The effective normal stress is computed by subtracting the pore water pressure form the total stress (i.e., $\sigma' = \sigma - u$). Thus, for the slip surface at a depth of 5 ft the effective stress is

$$\sigma' = 6834 - 6552 = 282 \text{ psf} \qquad (9.22)$$

and for the slip surface at a depth of 30 ft, the effective normal stress is 1691 psf (= 9803 − 8112). These represent the effective normal stresses after consolidation, σ'_{fc}. The effective normal stresses can also be calculated directly using submerged unit weights (γ') and the following equation, because there is no flow:

$$\sigma' = \gamma' z \cos^2\beta \qquad (9.23)$$

The shear stress on the slip surfaces is calculated from

$$\tau = (\gamma - \gamma_w)z \sin\beta \cos\beta \qquad (9.24)$$

For the slip surface at a depth of 5 ft, this gives

$$\tau = (125 - 62.4)(5)\sin(18.4°)\cos(18.4°) = 94 \text{ psf} \qquad (9.25)$$

and similarly, for the slip surface at a depth of 30 ft, the shear stress is 562 psf. These stresses represent the shear stresses, τ_{fc}, on the two potential slip surfaces after consolidation. The consolidation stresses are summarized in Table 9.2.

Second-stage analysis. For the second-stage analysis the undrained shear strengths are determined based on σ'_{fc} and τ_{fc}. The undrained shear strengths for the

Table 9.2 Summary of Stability Calculations for Rapid Drawdown of Example Slope

	Quantity	z = 5 ft	z = 30 ft
Stage 1	Total normal stress on the failure plane, σ_{fc}	6834 psf	9803 psf
	Pore water pressure, u	6552 psf	8112 psf
	Effective normal stress on the failure plane (consolidation pressure), σ'_{fc}	282 psf	1691 psf
	Shear stress on the failure plane after consolidation, τ_{fc}	94 psf	562 psf
Stage 2	Shear strength, τ_{ff} (K_c = 1)	2103 psf	2615 psf
	Shear strength, τ_{ff} ($K_c = K_f$)	237 psf	1419 psf
	Effective principal stress ratio for consolidation, K_1	2.0	2.0
	Undrained shear strength (interpolated), τ_{ff}	1585 psf	2283 psf
	Shear stress (after drawdown), τ	187 psf	1123 psf
	Factor of safety (undrained strengths)	8.48	2.03
Stage 3	Total normal stress (after drawdown), σ	563 psf	3376 psf
	Pore water pressure (after drawdown and drainage)	0	0
	Effective normal stress (after drawdown and drainage)	563 psf	3376 psf
	Drained shear strength	472 psf	2833 psf
Final stage	Governing strength after drawdown	472 psf (drained)	2283 psf (undrained)
	Factor of safety after drawdown	2.52 (third stage)	2.03 (second stage)

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$K_c = 1$ shear strength envelope are computed using the equation

$$\tau_{ff(K_c=1)} = d + \sigma'_{fc} \tan \psi \tag{9.26}$$

For the slip surface at a depth of 5 ft, this gives

$$\tau_{ff(K_c=1)} = 2000 + (282)\tan(20°) = 2103 \text{ psf} \tag{9.27}$$

The corresponding value for the slip surface at a depth of 30 ft is $\tau_{ff(K_c=1)} = 2615$ psf. The shear strengths from the envelope for $K_c = K_f$ are computed from

$$\tau_{ff(K_c=K_f)} = c' + \sigma'_{fc} \tan \phi' \tag{9.28}$$

For the slip plane at a depth of 5 ft, this gives

$$\tau_{ff(K_c=K_f)} = 0 + (282)\tan(40°) = 237 \text{ psf} \tag{9.29}$$

Similarly, for the slip surface at 30 ft depth, $\tau_{ff(K_c=K_f)}$ is 1419 psf.

The undrained shear strengths for the second-stage analysis are interpolated using the effective principal stress ratios and the undrained shear strength values determined above. The effective principal stress ratios after consolidation are calculated using the stresses from the first-stage analysis and Eq. (9.10). For the slip surface at a depth of 5 ft,

$$K_1 = \frac{282 + 94\{[\sin(40°) + 1]/\cos(40°)\}}{282 + 94\{[\sin(40°) - 1]/\cos(40°)\}} = 2.0 \tag{9.30}$$

The effective principal stress ratio for consolidation for the slip surface at a depth of 30 ft is also equal to 2.0.

The effective principal stress ratio at failure is calculated from Eq. (9.12). The effective principal stress ratio at failure is 4.6 and is the same for both depths because $c' = 0$. Undrained shear strengths are determined using Eq. (9.11). For the slip surfaces at depths of 5 and 30 ft, the shear strengths are as follows:

$$\tau_{ff} = \begin{cases} \dfrac{(4.6 - 2.0)2103 + (2.0 - 1)237}{4.6 - 1} = 1585 \text{ psf} \\ \quad \text{for 5-ft depth} \qquad (9.31) \\ \dfrac{(4.6 - 2.0)2615 + (2.0 - 1)1419}{4.6 - 1} = 2283 \text{ psf} \\ \quad \text{for 30-ft depth} \qquad (9.32) \end{cases}$$

The next step in the second-stage analysis is to calculate the factor of safety after drawdown using the undrained shear strengths. For the infinite slope and complete drawdown (no water above slope), the shear stress is calculated from

$$\tau = \gamma z \sin \beta \cos \beta \tag{9.33}$$

For the slip surface at a depth of 5 ft, this gives

$$\tau = (125)(5)\sin(18.4°)\cos(18.4°) = 187 \text{ psf} \tag{9.34}$$

Similarly, for the slip surface at a depth of 30 ft the shear stress is 1123 psf. The factors of safety for the slip surfaces are calculated by dividing the undrained shear strength by the shear stress. For the slip surfaces at depths of 5 and 30 ft, this gives, respectively,

$$F = \begin{cases} \dfrac{1585}{187} = 8.48 \qquad \text{for 5-ft depth} \qquad (9.35) \\ \dfrac{2283}{1125} = 2.03 \qquad \text{for 30-ft depth} \qquad (9.36) \end{cases}$$

These values represent the factors of safety for undrained conditions during drawdown.

Third-stage analysis. The third-stage analysis is begun by estimating the fully drained shear strengths of the soil (assuming that all excess pore water pressures due to drawdown have dissipated). For this example the water level is assumed to be lowered to such a depth that there will be no pore water pressures after drainage is complete. The total normal stress and effective stress are equal since the pore water pressures are zero. The effective stress for drained conditions after drawdown is computed from the equation

$$\sigma' = \gamma z \cos^2 \beta \tag{9.37}$$

For the slip surface at a depth of 5 ft, this gives

$$\sigma' = (125)(5)\cos^2(18.4°) = 563 \tag{9.38}$$

The drained shear strength is then

$$\tau_{\text{drained}} = 0 + (563)\tan(40°) = 472 \text{ psf} \tag{9.39}$$

This value (472 psf) is much less than the value of undrained shear strength (1585 psf) determined earlier. Therefore, the factor of safety would be lower if drainage occurred. The shear stress was calculated above [Eq. (9.34)] to be 187 psf, and the factor of safety for the slip surface at a depth of 5 ft is, therefore,

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$$F = \frac{472}{187} = 2.52 \quad (9.40)$$

For the slip surface at a depth of 30 ft, the effective normal stress after drainage is calculated as

$$\sigma' = (125)(30)\cos^2(18.4°) = 3376 \quad (9.41)$$

and the corresponding drained shear strength is

$$\tau_{\text{drained}} = 0 + (3376)\tan(40°) = 2833 \quad (9.42)$$

This value of 2833 psf for the drained shear strength is higher than the value of 1125 psf determined earlier for the undrained shear strength. Thus, the undrained shear strength controls, and the factor of safety is equal to the value of 2.02 that was calculated earlier [Eq. (9.36)].

The calculations for this example show that the drained shear strength controls the stability after drawdown for the shallow (5 ft) depth while the undrained shear strength controls the stability for deeper (30 ft) depth. It is commonly found that drained shear strength is smaller at shallower depths, and undrained shear strength is smaller at deeper depths. Generally, this pattern of drained and undrained shear strengths controlling the stability is expected to happen. For slip surfaces that encompass a range of depths, the controlling shear strength may be the drained strength in some parts and the undrained strength in others.

PARTIAL DRAINAGE

Partial drainage during drawdown may result in reduced pore water pressures and improved stability. Theoretically, such improvement in stability could be computed and taken into account by effective stress stability analyses. The computations would be performed like effective stress analyses for long-term stability except that pore water pressures for drawdown would be considered. Although such an approach seems logical, it is beyond the current state of practice and probably beyond the present state of the art. The principal difficulties lie in predicting the pore water pressures induced by drawdown. Approaches based on construction of flow nets, and most of the existing numerical solutions (finite difference, finite element), do not account for changes in pore water pressures induced by shear deformations. Ignoring shear-induced pore water pressures may lead to errors on the unsafe side. For a more complete discussion of procedures for estimating pore water pressures due to sudden drawdown, see Wright and Duncan (1987).

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